REINFORCED CONCRETE-I

(Design for Shear)

Design for Shear

The maximum shear V_u in a beam must not exceed the design shear capacity of the beam cross section ϕV_n, where $\phi = 0.75$ and V_n is the nominal shear strength of the concrete and the shear reinforcing.

$$\phi V_n \geq V_u$$

The vlaue of ϕV_n can be broken down into the design shear strength of the concrete ϕV_c plus design shear strength of the shear reinforcing ϕV_s. The value of ϕV_c is provided in the Code for different situations, and thus we are able to compute the required values of ϕV_s for each situation:

$$\phi V_n \geq V_u$$

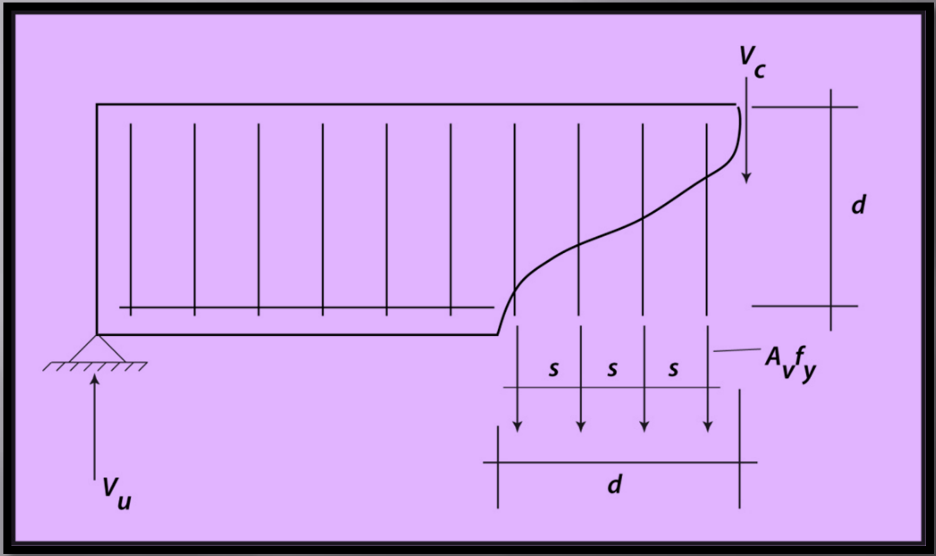
$$\Rightarrow \phi V_c + \phi V_s \geq V_u$$

For this derivation an equal sign is used:

$$V_u = \phi V_c + \phi V_s \qquad \text{where } V_c = \left(\frac{\sqrt{f_c'}}{6} \right) b_w d$$

ASSUMPTIONS

- Stirrups crossing the cracks have yielded.
- Shear crack form at a 45º angle.



A_v = Cross sectional area of the stirrup bar

n = number of stirrups crossing the crack

d = effective depth

s = center to center spacing of stirrups

$$V_s = A_v f_y n \quad \text{where } n = \frac{d}{s};$$

$$\therefore V_s = A_v f_y \frac{d}{s} \Rightarrow s = \frac{A_v f_y d}{V_s}$$

and the value of V_s can be determined as follows:

$$V_u = \phi V_c + \phi V_s \Rightarrow V_s = \frac{V_u - \phi V_c}{\phi}$$

For inclined stirrup:

$$V_s = \frac{A_v f_y (\sin\alpha + \cos\alpha) d}{s}$$

where α is the angle between the stirrups and the longitudinal axis of the member.

For a bent-up bar or group of bent-up bars at the same distance from the support:

$$V_s = A_v f_y \sin\alpha \le 3\sqrt{f_c'} b_w d$$

Code Requirements

1. If V_u exceeds one-half ϕV_c stirrups are required.
2. When shear reinforcement is required, ACI code specifies a minimum amount:

$$A_{v\,\min} = \frac{\sqrt{f_c'}\,b_w s}{16 f_y} \geq \frac{0.33 b_w s}{f_y}$$

3. To insure that every diagonal crack is crossed by at least one stirrup, the maximum spacing of stirrups is the smaller of $d/2$ or 600 mm.
4. If $V_s > \frac{1}{3}\sqrt{f_c'}\,b_w d$ these maximum spacings are to be reduced by one-half. These closer spacings will lead to narrower inclined cracks.
5. Under no circumstances may V_s be allowed to exceed $\frac{2}{3}\sqrt{f_c'}\,b_w d$. The shear strength of a beam cannot be increased indefinitely by adding more and more shear reinforcing because the concrete will eventually disintegrate no matter how much shear reinforcing is added.
6. In most cases, beam is to be designed for shear at a distnce d from the face of the support.

Summary of steps Involved in Vertical Stirrup Design

1. Draw V_u diagram
2. Calculate V_u at a distance d from the support (with certain exceptions)
3. Calculate ϕV_c where $V_c = \left(\frac{\sqrt{f_c'}}{6}\right) b_w d$
4. Stirrups are needed if $V_u > \frac{1}{2}\phi V_c$

Design of stirrups

1. Calculate theoretical stirrup spacing $s = \frac{A_v f_y d}{V_s}$ where $V_s = \frac{V_u - \phi V_c}{\phi}$

2. Determine maximum spacing to provide minimum area of shear reinforcement

$$A_{v\ \min} = \frac{\sqrt{f_c'} b_w s}{16 f_y} \text{ but not less than } A_{v\ \min} = \frac{0.33 b_w s}{f_y}$$

$$\Rightarrow s = \frac{16 f_y A_v}{\sqrt{f_c'} b_w} \text{ but not more than } \frac{f_y A_v}{0.33 b_w}$$

To insure that every diagonal crack is crossed by at least one stirrup

3. If $V_s < \frac{1}{3}\sqrt{f_c'} b_w d$: the maximum spacing of stirrups is the smaller of $d/2$ or 600 mm.

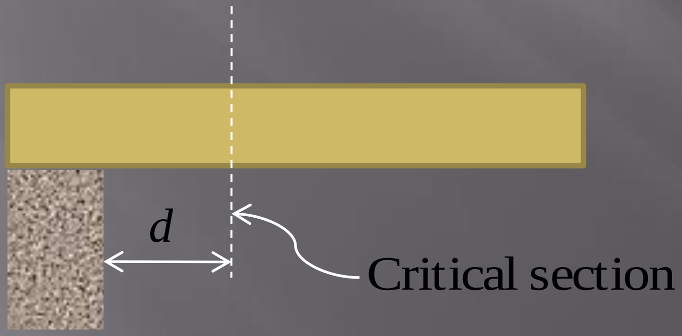
4. If $V_s > \frac{1}{3}\sqrt{f_c'} b_w d$: the maximum spacing of stirrups is the smaller of $d/4$ or 300 mm.

5. V_s may not be $> \frac{2}{3}\sqrt{f_c'} b_w d$

6. Minimum practical spacing $\approx$ 75 or 100 mm.

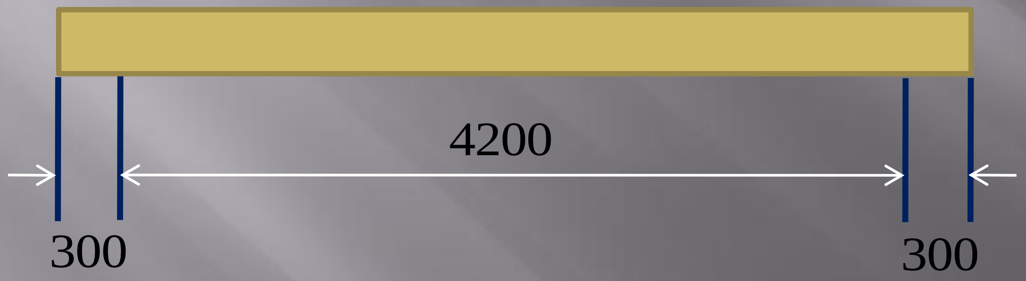
Critical Sections for Shear

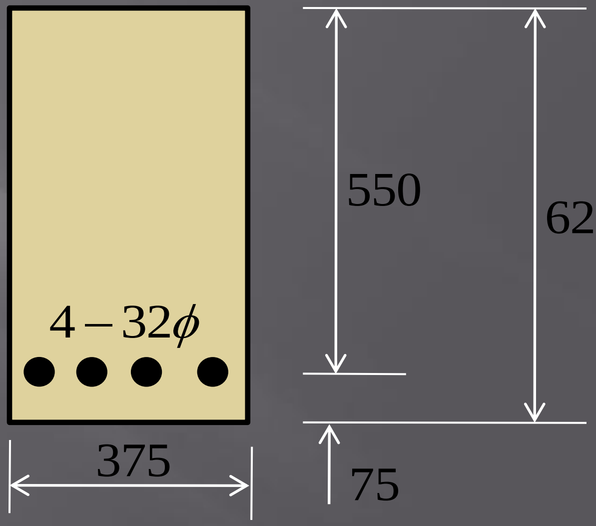
- When reaction in the direction of the applied shear introduces compression into end region of the member, sections located less than a distance *d* from the face of the support may be designed for the same shear as that computed at a distance *d*.



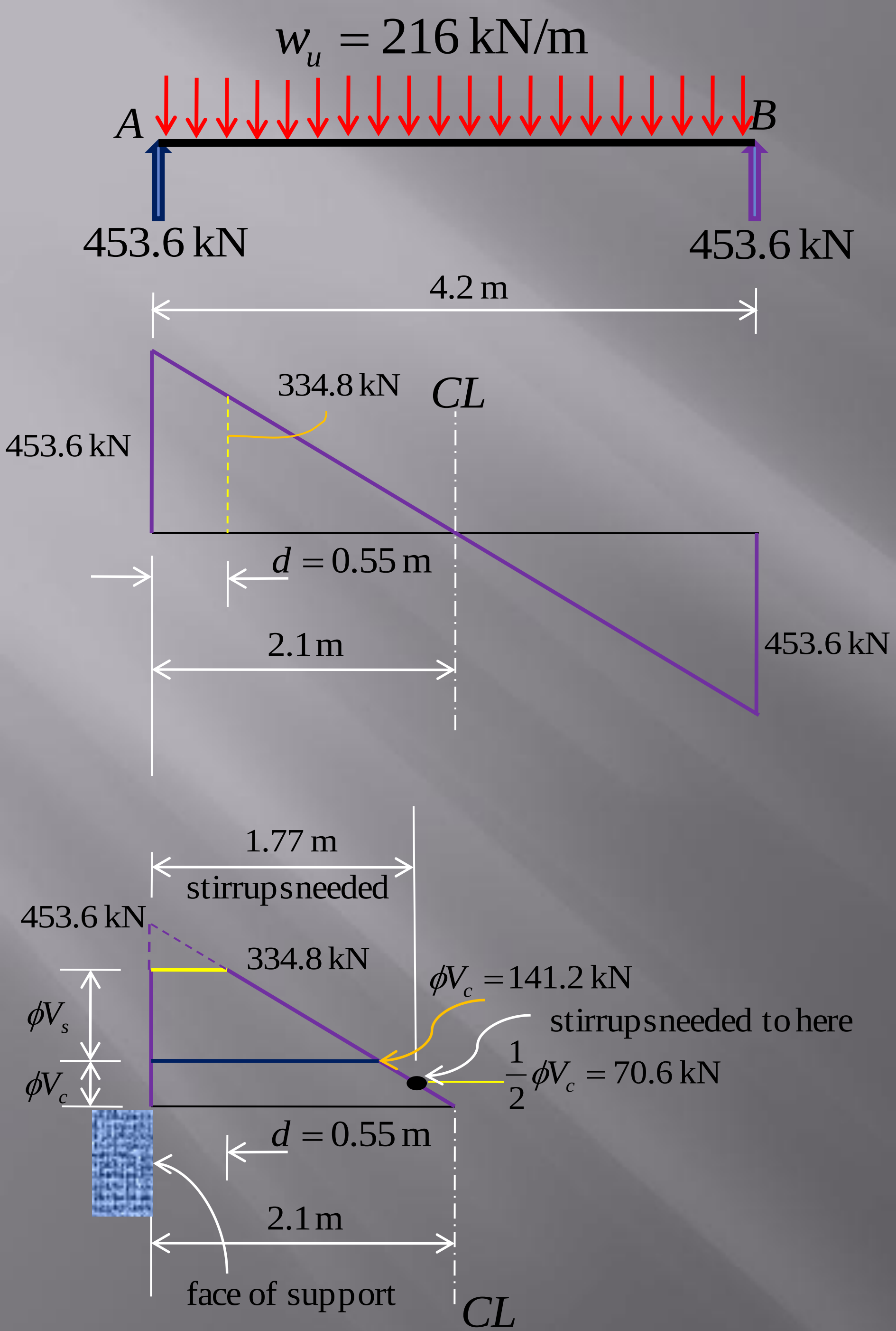
Problem

Calculate desired spacing for 10 mm two legged stirrups for the beam shown below. The beam has a clear span of 4.2 m and supports a dead load including its own weight of 60 kN/m and a live load of 90 kN/m. $f_c' = 30$ MPa., normal weight, and $f_y = 420$ MPa





All dimensions in mm



SOLUTION

$$w_u = 1.2DL + 1.6LL = 1.2\times 60 + 1.6\times 90 = 216 \text{ kN/m}$$

$$V_u \text{ at left end} = \frac{w_u L_c}{2} = \frac{216\times 4.2}{2} = 453.6 \text{ kN}$$

V_u at distance d from face of support:

$$\frac{V_u}{(2.1-d)} = \frac{453.6}{2.1} \Rightarrow \frac{V_u}{(2.1-0.55)} = \frac{453.6}{2.1}$$

$$\Rightarrow V_u = 334.8 \text{ kN}$$

$$\phi V_c = \phi\left(\frac{\sqrt{f_c'}}{6}\right)b_w d = 0.75\left(\frac{\sqrt{30}}{6}\right)375\times 550$$

$$\Rightarrow \phi V_c = 141209.7 \text{ N} = 141.2 \text{ kN} \qquad \therefore \frac{1}{2}\phi V_c = 70.6 \text{ kN}$$

$$\because V_u > \frac{1}{2}\phi V_c \Rightarrow \text{Stirrups are required.}$$

$$\therefore V_s = \frac{V_u - \phi V_c}{\phi} = \frac{334.8-141.2}{0.75} = 258.13 \text{ kN} \quad (\because V_u = \phi V_c + \phi V_s)$$

$$\frac{2}{3}\sqrt{f_c'}b_w d = \frac{2}{3}\sqrt{30}\times 375\times 550 = 753118.5 \text{ N} = 753.12 \text{ kN}$$

$$\because V_s < \frac{2}{3}\sqrt{f_c'}b_w d \quad \text{OK}$$

SOLUTION (Contd.)

Theoretical required spacing at left end :

$$s = \frac{A_v f_y d}{V_s} = \frac{2\times\left(\frac{\pi}{4}\times 10^2\right)\times 420\times 550}{(258.13\times 1000)} = 140.6\text{ mm}$$

let us provide stirrups @ 125 mm.

Length in which stirrups needed :

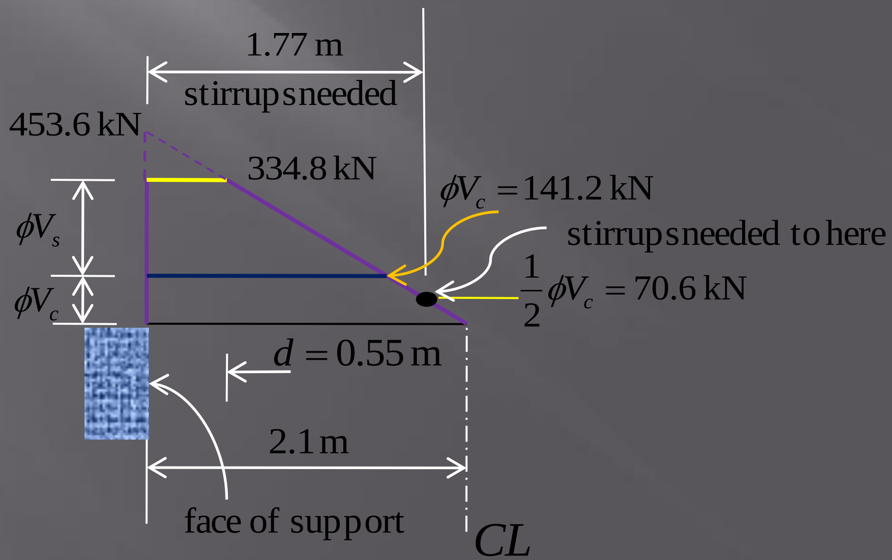
$$\frac{x}{70.6} = \frac{2.1}{453.6} \Rightarrow x = 0.33\text{ m or } (2.1 - 0.33) = 1.77\text{ m} \approx 1.75\text{ m from left end.}$$

Spacings selected : 2@175 mm (∵ 350/175 in that region where no spacing needed)

14@125 mm (∵ 1750/125 = 14)

Note stirrups are symmetric about the center line.

Note: for economy calculate different spacings according to variation of shear force.



SOLUTION (Contd.)

Maximum Spacing :

(i) To provide minimum area of shear reinforcement :

$$s_{\max} = \frac{16 f_y A_v}{\sqrt{f_c^{'}} b_w} = \frac{16 \times 420 \times 2 \times \left(\frac{\pi}{4} \times 10^2 \right)}{\sqrt{30} \times 375} = 513.9 \text{ mm}$$

$$\text{but not more than } \frac{f_y A_v}{0.33 b_w} = \frac{420 \times 2 \times \left(\frac{\pi}{4} \times 10^2 \right)}{0.33 \times 375} = 533.12 \text{ mm}$$

$$\Rightarrow \underline{\underline{s_{\max} = 513.9 \text{ mm}}}$$

(ii) To insure that every diagonal crack is crossed by at least one stirrup :

$$\frac{1}{3}\sqrt{f_c^{'}} b_w d = \frac{1}{3}\sqrt{30} \times 375 \times 550 = 376559.2 \text{ N} = 376.6 \text{ kN}$$

$$V_s = 258.13 \text{ kN}$$

$$s_{\max} = \text{Smaller of } \left(\frac{d}{2} \text{ or } 600 \text{ mm} \right) \qquad \because V_s < \frac{1}{3}\sqrt{f_c^{'}} b_w d$$

$$\Rightarrow s_{\max} = \text{Smaller of } (275 \text{ or } 600 \text{ mm}) = 275 \text{ mm}$$

$$\Rightarrow \underline{\underline{s_{\max} = 275 \text{ mm}}}$$

$$\because s < s_{\max} \quad \underline{\underline{\text{OK}}}$$